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Chapter 11
Fuzzy Logic and Data Mining in Disaster Mitigation

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Abstract Disaster mitigation and management is one of the most challenging examples of decision making under uncertain, missing, and sketchy, information. Even in the extreme cases where the nature of the disaster is known, preparedness plans are in place, and analysis, evaluation, and simulations of the disaster management procedures have been performed, the amount and magnitude of "surprises" that accompany the real disaster pose an enormous demand. In the more severe cases, where the entire disaster is an unpredicted event, the disaster management and response system might fast run into a chaotic state. Hence, the key for improving disaster preparedness and mitigation capabilities is employing sound techniques for data collection, information processing, and decision making under uncertainty. Fuzzy logic based techniques are some of the most promising approaches for disaster mitigation. The advantage of the fuzzy-based approach is that it enables keeping account on events with perceived low possibility of occurrence via low fuzzy membership/truth-values and updating these values as information is accumulated or changed. Several fuzzy logic based algorithms can be deployed in the data collection, accumulation, and retention stage, in the information processing phase, and in the decision making process. In this chapter a comprehensive assessment of fuzzy techniques for disaster mitigation is presented. The use of fuzzy logic as a possible tool for disaster management is investigated and the strengths and weaknesses of several fuzzy techniques are evaluated. In addition to classical fuzzy techniques, the use of incremental fuzzy clustering in the context of complex and high order fuzzy logic system is evaluated.

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11.1 Introduction

Disaster mitigation and management (DMM) is one of the most challenging examples of decision making under uncertain, missing, and sketchy, information. Even in the extreme cases where the nature of the disaster is known, preparedness plans are in place, and analysis, evaluation, and simulations of the disaster management procedures have been performed, the amount and magnitude of "surprises" that accompany the real disaster pose an enormous demand. In the more severe cases, where the entire disaster is an unpredicted event, the disaster management and response system might fast run into a chaotic state. Hence, the key for improving disaster preparedness and mitigation capabilities is employing sound techniques for data collection, information processing, and decision making under uncertainty.

Consequently, we must develop a formal set of tools to deal with the increasing number of potential disasters around the world on one hand and information dominance on the other hand. Moreover, we need a formal understanding of the relations between uncertainty, which is immersed in DMM programs, and fuzzy logic. Just as the formal notions of computer science have immensely benefited software and hardware design, a formal treatment of DMM programs should also lend to significant advances in the way that these programs and systems are developed and designed in the future.

Recently, the academic community and other agencies have presented spurring growth in the field of data mining in big-data systems. These advances are beginning to find their way into DMM programs and are redefining the way we address potential disaster and mitigate the effects of disasters. Nevertheless, academia, industry, and governments need to engage as a unified entity to advance new technologies as well as applied established technologies in preparation and response to the specific emerging problems of disasters. Since research and development is seldom conducted in national isolation it must include international collaboration and global activities. This will bring many countries and scientists together in order to enjoy broad access to all leading edge technologies and tools to be used for prevention and remedies of disasters.

Analysis of disasters shows three types of challenges, the first is the ability to predict the occurrence of disasters, the second is the need to produce a preparedness plan, and the third is the actual real time response activities related to providing remedies for a currently occurring disaster. Inspired by the metaphor of black swan coined by Taleb [1, 2] we show that every disaster falls into the category of a gray level swan (with black swan being a special case of gray level swan). Furthermore, we show that each of the three challenges involves dealing with uncertainty and can be addressed via fuzzy logic based tools and techniques.

Experience shows that swans come in different shades of gray, spawning the range from black-to-white. There is no need to expand on white swans. Those are adorable animals and under the Taleb metaphor, they represent pleasant events, highly predictable pleasant events, or pleasant surprises. Our collective knowledge, however, tells us that white swans are rare. Moreover, sometimes white swans change abruptly without advanced notice into black or gray swans. The black swans are the other extreme. They represent completely unpredictable highly adverse events (disasters). Again, common experience shows that black swans are rare. This is due to two reasons: (1) there is almost always some information about the upcoming black swan. This is evident from the term "connecting the dots," which refers to the fact that information about disasters might be available; but, our ability to process this information is limited. The second is related to the authors' personal observation that often there is some amount of "good news" that accompanies any type of "bad news." Following these observation we classify all the swans as gray level swans; where the level of gray is related to the amount of surprise and the severity of the disastrous event represented by the swan. We further elaborate on gray swans in Sect. 11.2. In Sect. 11.3, we review methods for improving our ability to identify and "bleach" gray swans using fuzzy logic-based tools.

Predicting the encounter with gray swans is an important component of DMM. It can enable early setting of a mitigation plan. Nevertheless, the fact that a disaster (gray swan) is somewhat predictable does not completely reduce the amount of surprise that accompany the actual occurrence of the disaster. Hence, any mitigation plan; that is, a set of procedures compiled in order to address the adverse effects of disasters, should be flexible enough to handle additional surprises. We refer to these surprises as second generation swans. Finally, the real time ramification program is the actual set of procedures enacted and executed as the disaster occurs. There are two preconditions for successful remedy of the disaster effect. First, the leadership of the authorities who have to attempt following the original mitigation plan as close as possible while instilling a sense of trust and calmness in the people that experience the disaster and the mitigation provider teams. Second, and as a part of their leadership traits, the authorities must possess the ability to adapt their remedy procedures to the dynamics of the disaster; potentially providing effective improvisations. Again, this relates to the notion of second generation gray swans which are secondary disastrous events that evolve from the main disaster. A simple example for such swans is events of looting and violence that might accompany a major disaster. For this end, fast automatic assessment of the dynamics is paramount. Again, numerous tools including fuzzy logic based tools can assist the leaders in getting a good grasp on the disaster dynamics. A related notion is the notion of unknown unknowns (dark black swans) and known unknowns (gray swans).

Fuzzy logic based techniques are some of the most promising approaches for disaster mitigation. The advantage of the fuzzy-based approach is that it enables keeping account on events with perceived low possibility of occurrence via low fuzzy membership/truth-values and updating these values as information is accumulated or changed. Several fuzzy logic based algorithms can be deployed in the data collection, accumulation, and retention stage, in the information processing phase, and in the decision making process. In this chapter a comprehensive assessment of fuzzy techniques for disaster mitigation is presented. The use of fuzzy logic as a possible tool for disaster management is investigated and the strengths and weaknesses of several fuzzy techniques are evaluated. In addition to classical fuzzy techniques, the use of incremental fuzzy clustering in the context of complex and high order fuzzy logic system is evaluated.

The rest of the chapter elaborates on the unknown unknowns (that is, black swans) and known unknowns (i.e., gray swans) their relation to DMM and uncertainty which can be addressed via fuzzy logic based tools. This is elaborated in Sect. 11.2. Section 11.3, provides an overview of several fuzzy logic based tools for dealing with the uncertainty of the two sorts of unknowns and Sect. 11.4 provides brief conclusions as well as proposals for future enhancements of this research.

11.2 Uncertainty in Disaster Mitigation and Management

Disasters occur with different degrees of unpredictability and severity which is manifested in two main facets. First, the actual occurrence of the disaster might be difficult (potentially impossible) to predict. Second, regardless of the level predictability of the disaster, it is very likely that the disaster will be accompanied by secondary effects. Hence, disasters are a major source of "surprise" and uncertainty and their mitigation and management requires sound automatic and intelligent handling of uncertainty. Often, the stakeholders of DMM programs are classifying the unpredictability of disasters as two types of unknowns: unknown unknowns and known unknowns.

11.2.1 Unknown Unknowns and Known Unknowns

In his excellent books [1, 2], Nassim Nicholas Taleb describes the features of black swan events. We provide a slightly different set of features.

1. The black swan is an outlier; lying outside of the space of our regular expectations.
2. It has very low predictability and it carries an extremely adverse impact
3. The unknown component of the event is far more relevant than the known component.
4. Finally, we can explain, de facto, the event and through those explanations make it predictable in retrospect.

In this sense, the black swan represents a class of problems that can be referred to as the "unknown unknowns." However, a thorough investigation of many of the events that are widely considered as black swans, e.g., the September 11, 2001 attack in NYC, shows that there was available information concerning the attack; yet, this information did not affect the decision making and response prior to the attack.

This suggests that the term black swan is a bit too extreme and one should consider using the term gray swan. In this chapter, a gray swan represents an unlikely event that can be anticipated and carries an extremely adverse impact. In this respect the gray swans represent a two dimensional spectrum of information. The first dimension represents the predictability of the event where black swans are highly unpredictable and white swans are the norm. The second dimension represents the amount of adverse outcome embedded in the event; with black swans representing the most adverse outcomes. Consequently, the black swan is a special case of a gray swan. The following is a partial list of well-known gray swans each of which has a different degree of surprise as well as different degree of severity.

1. Bangladeshi factory collapse; Iceland volcano eruption
2. Fukushima – tsunami followed by nuclear radiation and risk of meltdown of nuclear facilities in the area.
3. 9/11 NYC attack followed by the collapse of the Twins
4. Y2K (was this a wolf-wolf-wolf ignited by grid?)
5. Financial Markets (1987, 2008), Madoff
6. 1998 Long-Term Capital Management
7. Yom Kippur War
8. December 7, 1941 - Pearl Harbor
9. Lincoln, Kennedy, Sadat, and Rabin assassinations

One type of gray swans relates to a set of events that can be considered as known unknowns. For example, a hurricane occurring in Florida during the hurricane season should not be considered as a part of the set of unknown unknowns. It should not surprise the responsible authorities. Moreover, often, there is a span of a few days between the identification of the hurricane and the actual landfall. Regardless, even a predictable hurricane land-fall carries numerous secondary disastrous events that are hard to predict.

We refer to these secondary events as second generation gray swans. Second generation swans are generated and/or detected while the disaster is in effect. A gray swan might (and according to the Murphy lows is likely to) spawn additional gray swans. Generally, second generation swans evolve late and fast. Hence, they introduce a more challenging detection problem and require specialized identification tools such as dynamic clustering.

11.2.2 Preparedness and Real Time Ramification

Following the discussion above, disaster mitigation and management has two key components. The first is referred to as preparedness. In preparedness, authorities consider as many known unknowns as possible, and compile procedures for early prediction, detection, and alert of disasters as well as sound remedy procedures for these disasters. The second component of DMM is referred to as real time remedies.

Considering real time remedies, there is no recipe (algorithm) for success. One can note that *leadership* is paramount and trying to adhere to a set of "best practices" which have been set ahead of time and used in training and ramification exercises can help reducing cost and risk. Nevertheless, a great amount of ability to improvise, rapidly change plans as dictated by changing circumstances, and significant amount of flexibility is required.

Even in the extreme cases where the nature of the disaster is known, preparedness plans are in place, and analysis, evaluation, and simulations of the disaster management procedures have been performed, the amount and magnitude of "surprises" that accompany the real disaster pose an enormous demand. Detecting relatively slow evolving [first generation] gray swans before the disaster occurs and relatively fast evolving second generation gray swans requires an adequate set of uncertainty management tools. In addition, existing swans might (are likely to) spawn unanticipated second generation swans. The collapse of the Twins in 9/11 is an example of a second generation gray swan.

Fuzzy logic is one of the suggested tools that can help creating a better understanding of DMM tools including, but no limited to intelligent robotics, learning and reasoning, language analysis and understanding, and data mining. While NATO continues to lead a strong technical agenda in DMM technologies, academia and industry must assume a preeminent position in driving a variety of leading-edge technologies and tools in order to address and mitigate disasters. We believe that the role of our research in fuzzy logic and uncertainty management in achieving these goals is critical for producing a successful DMM programs.

This emerging field must therefore drive a novel set of research directions for the USA and NATO assisting the scientific community and the private sector to develop a science and tools for anti-terror management.

11.3 Tools for Predictions and Evaluations of Fuzzy Events

Fuzzy logic based techniques are some of the most promising approaches for disaster mitigation. The advantage of the fuzzy-based approach is that it enables keeping account on events with perceived low possibility of occurrence via low fuzzy membership/truth-values and updating these values as information is accumulated or changed. Several fuzzy logic based algorithms can be deployed in the data collection, accumulation, and retention stage, in the information processing phase, and in the decision making process. Therefore, in this section we describe several possible fuzzy tools to try and predict disasters and cope with evolving disasters via sound DMM programs. We consider the following fuzzy logic based tools:

1. Fuzzy Switching Mechanisms
2. Fuzzy Expected Value
3. Fuzzy Relational Data Bases, Fuzzy Data-Mining and Fuzzy social Network Architectures (FSNA)

4. Complex and Multidimensional Fuzzy Sets, Logic, and Systems
5. Neuro-Fuzzy-Based Logic, and Systems
6. Dynamic and Incremental Fuzzy Clustering

11.3.1 Making Decisions with No Data

As an example for this idea we will use the fuzzy treatment of the transient behavior of a switching system and its static hazards [3]. Perhaps the major reason for the ineffectiveness of classical techniques in dealing with static hazard and obtaining a logical explanation of the existence of static hazard lies in their failure to come to grips with the issue of fuzziness. This is due to the fact that the hazardous variable implies imprecision in the binary system, which does not stem from randomness; but, from a lack of sharp transition between members in the class of input states. Intuitively, fuzziness is a type of imprecision which stems from a grouping of elements into classes that do not have sharply defined boundaries - that is, in which there is no sharp transition from membership to non-membership. Thus, the transition of a state has a fuzzy behavior during the transition time, since this is a member in an ordered set of operations, some of which are fuzzy in nature.

Any fuzzy-valued switching function can be expressed in disjunctive and conjunctive normal forms, in a similar way to two-valued switching functions. As before, fuzzy-valued switching functions over n variables can be represented by the mapping $f:[0,1]^n \rightarrow [0,1]$. We define a V-fuzzy function as a fuzzy function $f(x)$ such that $f(\xi)$ is a binary function for every binary n-dimensional vector ξ. It is clear that a V-fuzzy function f induces a binary function F such that $F:[0,1]^n \rightarrow [0,1]$ determined by $F(\xi)=f(\xi)$ for every binary n-dimensional vector ξ.

If the B-fuzzy function f describes the complete behavior of a binary combinational system, its steady-state behavior is represented by F, the binary function induced by f. Let $f(x)$ be an n-dimensional V-fuzzy function, and let ξ and ρ be adjacent binary n-dimensional vectors. The vector $T_{\xi_j}^{\rho}$ is a static hazard of f iff $f(\xi) = f(\rho) \neq f\left(T_{\xi_j}^{\rho}\right)$.

If $f(\xi)=f(\rho)=1$, $T_{\xi_j}^{\rho}$ is a 1-hazard. If $f(\xi)=f(\rho)=0$, $T_{\xi_j}^{\rho}$ is a 0-hazard. If f is B-fuzzy and $T_{\xi_j}^{\rho}$ is a static hazard, then $f\left(T_{\xi_j}^{\rho}\right)$ has a perfect fuzzy value, that is, $f\left(T_{\xi_j}^{\rho}\right) \in (0,1)$. Consider the static hazard as a malfunction represented by an actual or potential deviation from the intended behavior of the system. We can detect all static hazards of the V-fuzzy function $f(x)$ by considering the following extension of Shannon normal form. Let $f(\overline{x})$, $\overline{x}=(x_1, x_2, \ldots, x_n)$, be a fuzzy function and denote the vector

$$\left(x_1, x_2, x_{j-1}, x_{j+1}, \ldots, x_n,\right) \text{by } x^j,$$

By successive applications of the rules of fuzzy algebra, the function $f(x)$ may be expanded about, say, x_j as follows:

$$f(x) = x_j f_1\left(x^j\right) + \overline{x}_j f_2\left(x^j\right) + x_j\overline{x}_j f_3\left(x^j\right) + f_4\left(x^j\right),$$

where f_1, f_2, f_3, and f_4 are fuzzy functions. It is clear that the same expansion holds when the fuzzy functions are replaced by B-fuzzy functions of the same dimension. Let ξ and ρ be two adjacent n-dimensional binary vectors that differ only in their j th component. Treating ξ_j as a perfect fuzzy variable during transition time implies that $T_{\xi_j}^{\rho}$ is a 1-hazard of f iff $f(\xi) = f(\rho) = 1$ and $f\left(T_{\xi_j}^{\rho}\right) \in [0, 1)$. We show that the above conditions for the vector $T_{\xi_j}^{\rho}$ to be 1-hazard yield the following result.

Theorem 1 ([3]): The vector $T_{\xi_j}^{\rho}$ is a 1-hazard of the B-fuzzy function $f(x)$ given above iff the binary vector ξ_j is a solution of the following set of Boolean equations:

$$f_1\left(x^j\right) = 1, \quad f_2\left(x^j\right) = 1, \quad f_4\left(x^j\right) = 0.$$

Proof

State 1: $\xi_j = 1$ and $\overline{\xi}_j = 0$ imply $f_1(\xi^j) + f_4(\xi^j) = 1$.
State 2: $\xi_j = 0$ and $\overline{\xi}_j = 1$ imply $f_2(\xi^j) + f_4(\xi^j) = 1$.

Transition state: $\xi_j \in (0, 1)$ [which implies $\overline{\xi}_j \in (0, 1)$], and thus:

$$0 \leq max\left\{min\left[\xi_j, f_1\left(\xi^j\right)\right], min\left[\overline{\xi}_j, f_2\left(\xi^j\right)\right], min\left[\xi_j, \overline{\xi}_j, f_3\left(\xi^j\right)\right], f_4\left(\xi^j\right)\right\} < 1.$$

It is clear from the transition state that $f_4(\xi_j)$ cannot be equal to one, and thus:

$$f_4\left(\xi^j\right) = 0, \quad f_1\left(\xi^j\right) = f_2\left(\xi^j\right) = 1.$$

Several items must be pointed out. The system is not a fuzzy system. It is a Boolean system. The modeling of the system as a fuzzy system, due to the lack of knowledge regarding the behavior of x_j during the transition provided us with a tool to make decisions (regarding the Boolean values of f_1, f_2 and f_4) with no data whatsoever regarding x^j. Thus, we were able to make non-fuzzy decisions in a deterministic environment with no data. The interesting question is whether or not we can apply this idea to DMM programs.

11.3.2 Fuzzy Expectations

Ordinarily, imprecision and indeterminacy are considered to be statistical, random characteristics and are taken into account by the methods of probability theory.

In real situations, a frequent source of imprecision is not only the presence of random variables, but the impossibility, in principle, of operating with exact data as a result of the complexity of the system, or the imprecision of the constraints and objectives. At the same time, classes of objects that do not have clear boundaries appear in the problems; the imprecision of such classes is expressed in the possibility that an element not only belongs or does not belong to a certain class, but that intermediate grades of membership are also possible. The membership grade is subjective; although it is natural to assign a lower membership grade to an event that have a lower probability of occurrence. The fact that the assignment of a membership function of a fuzzy set is "non-statistical" does not mean that we cannot use probability distribution functions in assigning membership functions. As a matter of fact, a careful examination of the variables of fuzzy sets reveals that they may be classified into two types: statistical and non-statistical.

Definition 1 ([3]) Let B be a Borel field (σ -algebra) of subsets of the real line Ω. A set function $\mu(\cdot)$ defined on B is called a fuzzy measure if it has the following properties:

1. $\mu(\Phi) = 0$ (Φ is the empty set);
2. $\mu(\Omega) = 1$;
3. If $\alpha, \beta \in B$; with $\alpha \subset \beta$, then $\mu(\alpha) \leq \mu(\beta)$;
4. If $\{\alpha_j | 1 \leq j < \infty\}$ is a monotone sequence, then

$$\lim_{j\to\infty} \left[\mu\left(\alpha_j\right)\right] = \mu\left[\lim_{j\to\infty}\left(\alpha_j\right)\right].$$

Clearly, $\Phi, \Omega \in B$; also, if $\alpha_j \in B$ and $\{\alpha_j | 1 \leq j < \infty\}$ is a monotonic sequence, then $\lim_{j\to\infty}\left(\alpha_j\right) \in B$. In the above definition, (1) and (2) mean that the fuzzy measure is bounded and nonnegative, (3) means monotonicity (in a similar way to finite additive measures used in probability), and (4) means continuity. It should be noted that if Ω is a finite set, then the continuity requirement can be deleted. (Ω, B, μ) is called a fuzzy measure space; $\mu(\cdot)$ is the fuzzy measure of (Ω, B). The fuzzy measure μ is defined on subsets of the real line. Clearly, $\mu[\chi_A \geq T]$ is a non-increasing, real-valued function of T when χ_A is the membership function of set A. Throughout our discussion, we use ξ^T to represent $x | \chi_A(x) \geq T$ and $\mu(\xi^T)$ to represent $\mu[\chi_A \geq T]$, assuming that the set A is well specified. Let $\chi A : \Omega \to [0,1]$ and $\xi^T = x | \chi_A(x) \geq T$. The function χ_A is called a B-measurable function if $\xi^T \in B, \forall\, T \in [0,1]$. Definition 2 defines the fuzzy expected value (*FEV*) of χ_A when $\chi_A \in [0,1]$. Extension of this definition when $\chi_A \in [a,b], a < b < \infty$, is presented later.

Definition 2 ([3]): Let χ_A be a B -measurable function such that $\chi_A \in [0,1]$. The fuzzy expected value (*FEV*) of χ_A over a set A, with respect to the measure

$\mu(\cdot)$, is defined as $\sup_{T\in[0,1]}\left\{min\left[T, \mu\left(\xi^T\right)\right]\right\}$, where $\xi^T = \{x|\chi_A(x) \geq T\}$. Now, $\mu\{x|\chi_A(x) \geq T\} = f_A(T)$ is a function of the threshold T. The actual calculation of $FEV(\chi_A)$ then consists of finding the intersection of the curves $T = f_A(T)$. The intersection of the two curves will be at a value $T = H$, so that $FEV(\chi_A = H \in [0, 1]$. It should be noted that when dealing with the $FEV(\eta)$ where $\eta \in [0, 1]$, we should not use a fuzzy measure in the evaluation but rather a function of the fuzzy measure, η', which transforms η under the same transformation that χ and T undergo to η and T', respectively. In general the FEV has the promise and the potential to be used as a very powerful tool in developing DMM technologies.

11.3.3 Fuzzy Relational Databases and Fuzzy Social Network Architecture

The Fuzzy Relational Database (FRDB) model which is based on research in the fields of relational databases and theories of fuzzy sets and possibility is designed to allow representation and manipulation of imprecise information. Furthermore, the system provides means for "individualization" of data to reflect the user's perception of the data [4]. As such, the FRDB model is suitable for use in fuzzy expert system and other fields of imprecise information-processing that model human approximate reasoning such as FSNA [5, 6].

The objective of the FRDB model is to provide the capability to handle imprecise information. The FRDB should be able to retrieve information corresponding to natural language statements as well as relations in FSNA. Although most of these situations cannot be solved within the framework of classical database management systems, they are illustrative of the types of problem that human beings are capable of solving through the use of approximate reasoning. The FRDB model and the FSNA model retrieve the desired information by applying the rules of fuzzy linguistics to the fuzzy terms in the query.

The FRDB as well as the FSNA development [4–6] were influenced by the need for easy-to-use systems with sound theoretical foundations as provided by the relational database model and theories of fuzzy sets and possibility. They address the following issues:

1. representation of imprecise information,
2. derivation of possibility/certainty measures of acceptance,
3. linguistic approximations of fuzzy terms in query languages,
4. development of fuzzy relational operators (IS, AS . . . AS, GREATER, . . .),
5. processing of queries with fuzzy connectors and truth quantifiers,
6. null-value handling using the concept of the possibilities expected value,
7. modification of the fuzzy term definitions to suit the individual user.

The fuzzy relational data base and the FSNA are collections of fuzzy time-varying relations which may be characterized by tables, graphs, or functions, and manipulated by recognition (retrieval) algorithms or translation rules.

As an example let us take a look at one of these relations, the similarity relation. Let D_i be a scalar domain, $\in D_i$. Then $s(x, y) \in [0, 1]$ is a similarity relation with the following properties: Reflexivity: $s(x, x) = 1$; Symmetry: $s(x, y) = s(y, x)$; Θ -transitivity: where Θ is most commonly specified as max-min transitivity. If, $y, z \in U$, then $s(x, z) \geq max(y \in D_i)\ \ min(s(x, y), s(y, z))$. Another example is the proximity relation defined below. Let D_i be a numerical domain and, $y, z \in D_i$. Here $p(x, y) \in [0, 1]$ is a proximity relation that is reflexive, and symmetric with transitivity of the form

$$p\,(x, z) \geq max\,(y \in D_i)\ \ p\,(x, y) * p\,(y, z)\,.$$

The generally used form of the proximity relations is $p(x, y).\, e^{-\beta|x-y|}$, where $\beta > 0$. This form assigns equal degrees of proximity to equally distant points. For this reason, it is referred to as absolute proximity in the FRDB and FSNA models. Similarity and proximity are used in evaluation of queries of the general form: “Find X such that $X.\,A \ominus d$” Where $X.\,A$ is an attribute of $X, d \in D$ is a value of attribute A defined on the domain D, and $\ominus$ is a fuzzy relational operator. Clearly both FRDS and FSNA may have numerous applications in black swan as well as gray swan prediction.

In many DMM programs and disaster models the amount of information is determined by the amount of the uncertainty – or, more exactly, it is determined by the amount by which the uncertainty has been reduced; that is, we can measure information as the decrease of uncertainty. The concept of information itself has been implicit in many DMM models. This is both as a substantive concept important in its own right and as a consonant concept that is ancillary to the entire structure of DMM

11.3.4 Complex Fuzzy Membership Grade

Several aspects of the DMM program can utilize the concept of complex fuzzy logic [3, 7–14]. Complex fuzzy logic can be used to represent the two dimensional information embedded in the description of a disaster; namely, the severity and uncertainty. In addition, inference based on complex fuzzy logic can be used to exploit the fact that variables related to the uncertainty that it a part of disasters is multi-dimensional and cannot be readily defined via single dimensional clauses connected by single dimensional connectives. Finally, the multi-dimensional fuzzy space defined as a generalization of complex fuzzy logic can serve as a media for clustering of disaster in a linguistic variable-based feature space.

Tamir et al. introduced a new interpretation of complex fuzzy membership grade and derived the concept of pure complex fuzzy classes [13]. This section introduces the concept of a pure complex fuzzy grade of membership, the interpretation of this concept as the denotation of a fuzzy class, and the basic operations on fuzzy classes.

To distinguish between classes, sets, and elements of a set we use the following notation: a class is denoted by an upper case Greek letter, a set is denoted by an upper case Latin letter, and a member of a set is denoted by a lower case Latin letter.

The Cartesian representation of the pure complex grade of membership is given in the following way:

$$\mu\left(V, z\right) = \mu_r(V) + j\mu_i(z)$$

Where $\mu_r(V)$ and $\mu_i(z)$, the real and imaginary components of the pure complex fuzzy grade of membership, are real value fuzzy grades of membership. That is, $\mu_r(V)$ and $\mu_i(z)$ can get any value in the interval $[0, 1]$. The polar representation of the pure complex grade of membership is given by:

$$\mu\left(V, x\right) = r(V)e^{j\sigma\phi(z)}$$

Where $r(V)$ and $\phi(z)$, the amplitude and phase components of the pure complex fuzzy grade of membership, are real value fuzzy grades of membership. That is, they can get any value in the interval $[0, 1]$. The scaling factor, σ is in the interval $(0, 2\pi]$. It is used to control the behavior of the phase within the unit circle according to the specific application. Typical values of σ are $\left\{1, \frac{\pi}{2}, \pi, 2\pi\right\}$. Without loss of generality, for the rest of the discussion in this section we assume that $\sigma = 2\pi$.

The difference between pure complex fuzzy grades of membership and the complex fuzzy grade of membership proposed by Ramot et al. [11, 12], is that both components of the membership grade are fuzzy functions that convey information about a fuzzy set. This entails different interpretation of the concept as well as a different set of operations and a different set of results obtained when these operations are applied to pure complex grades of membership. This is detailed in the following sections.

11.3.4.1 Complex Fuzzy Class

A fuzzy class is a finite or infinite collection of objects and fuzzy sets that can be defined in an unambiguous way and complies with the axioms of fuzzy sets given by Tamir et al. and the axioms of fuzzy classes given by *Běhounek* [9, 15–20]. While a general fuzzy class can contain individual objects as well as fuzzy sets, a *pure fuzzy class of order one* can contain only fuzzy sets. In other words, individual objects cannot be members of a pure fuzzy class of order one. A pure fuzzy class of order M is a collection of pure fuzzy classes of order $M - 1$. We define a *Complex Fuzzy Class* Γ to be a pure fuzzy class of order one i.e., a fuzzy set of fuzzy sets. That is,

$\Gamma = \{V_i\}_{i=1}^{\infty}$; or $\Gamma = \{V_i\}_{i=1}^{N}$ where V_i is a fuzzy set and N is a finite integer. Note that despite the fact that we use the notation $\Gamma = \{V_i\}_{i=1}^{\infty}$ we do not imply that the set of sets $\{V_i\}$ is enumerable. The set of sets $\{V_i\}$ can be finite, countably infinite, or uncountably infinite. The use of the notation $\{V_i\}_{i=1}^{\infty}$ is just for convenience.

The class Γ is defined over a universe of discourse T. It is characterized by a pure complex membership function $\mu_\Gamma(V,z)$ that assigns a complex-valued grade of membership in Γ to any element $z \in U$ (where U is the universe of discourse). The values that $\mu_\Gamma(V,z)$ can receive lie within the unit square or the unit circle in the complex plane, and are in one of the following forms:

$$\mu_\Gamma(V,z) = \mu_r(V) + j\mu_i(z)$$

$$\mu_\Gamma(z,V) = \mu_r(z) + j\mu_i(V)$$

Where $\mu_r(\alpha)$ and $\mu_i(\alpha)$, are real functions with a range of [0,1]. Alternatively:

$$\mu_\Gamma(V,z) = r(V)e^{j\theta\phi(z)}$$
$$\mu_\Gamma(z,V) = r(z)e^{j\theta\phi(V)}$$

Where $r(\alpha)$ and $\phi(\alpha)$, are real functions with a range of [0, 1] and $\theta \in (0, 2\pi]$.

In order to provide a concrete example we define the following pure fuzzy class. Let the universe of discourse be the set of all the hurricanes that hit the East Coast of the USA (in any time in the past) along with a set of attributes related to hurricanes such as wind speed, rain, movement of the hurricane eye, and related surges. Let M_i denote the set of hurricanes that hit the East Coast of the USA in the last i years. Furthermore consider a function (f_1) that associates a number between 0 and 1 with each set of hurricanes. For example, this function might reflect the severity in terms of average wind gust of all the hurricanes in the set. In addition, consider a second function (f_2) that associates a number between 0 and 1 with each specific hurricane. For example, this function might be a normalized value of level of destructiveness of the hurricane. The functions (f_1,f_2) can be used to define a pure fuzzy class of order one. A compound of the two functions in the form of a complex number can represent the degree of membership in the pure fuzzy class of "destructive" (e.g., catastrophic) hurricanes in the set of hurricanes that occurred in the last 10 years.

Formally, let U be a universe of discourse and let 2^U be the power set of U. Let f_1 be a function from 2^U to [0, 1] and let f_2 be a function that maps elements of U to the interval [0, 1]. For $V \in 2^U$ and $z \in U$ define $\mu_\Gamma(V,z)$ to be:

$$\mu_\Gamma(V,z) = \mu_r(V) + j\mu_i(z) = f_1(V) + jf_2(z)$$

Then, $\mu_\Gamma(V,z)$ defines a pure fuzzy class of order one, where for every $V \in 2^U$, and for every $z \in U$,

$\mu_\Gamma(V, z)$; is the degree of membership of z in V and the degree of membership of V in Γ. Hence, a complex fuzzy class Γ can be represented as the set of ordered triples:

$$\Gamma = \left\{ V, z, \mu_\Gamma(V, z) \middle| V \in 2^U, z \in U \right\}$$

Depending on the form of $\mu_\Gamma(\alpha)$ (Cartesian or polar), $\mu_r(\alpha)$, $\grave{\imath}_i(\alpha)$, $r(\alpha)$, and $\phi(\alpha)$ denote the degree of membership of z in V and/ or the degree of membership of V in Γ. Without loss of generality, however, we assume that $\mu_r(\alpha)$ and $r(\alpha)$ denote the degree of membership of V in Γ for the Cartesian and the polar representations respectively. In addition, we assume that $\mu_i(\alpha)$ and $\phi(\alpha)$ denote the degree of membership of z in V for the Cartesian and the polar representations respectively. Throughout this chapter, the term complex fuzzy class refers to a pure fuzzy class with pure complex-valued membership function, while the term fuzzy class refers to a traditional fuzzy class such as the one defined by *Bĕhounek* [15].

Degree of Membership of Order *N*

The traditional fuzzy grade of membership is a scalar that defines a fuzzy set. It can be considered as degree of membership of order 1. The pure complex degree of membership defined in this chapter is a complex number that defines a pure fuzzy class. That is, a fuzzy set of fuzzy sets. This degree of membership can be considered as degree of membership of order 2 and the class defined can be considered as a pure fuzzy class of order 1. Additionally, one can consider the definition of a fuzzy set (a class of order 0) as a mapping into a one dimensional space and the definition of a pure fuzzy class (a class of order 1) as a mapping into a two dimensional space. Hence, it is possible to consider a degree of membership of order N as well as a mapping into an N -dimensional space. The following is a recursive definition of a fuzzy class of order N. Note that part 2 of the definition is not really necessary it is given in order to connect the terms pure complex fuzzy grade of membership and the term grade of membership of order 2.

Definition 3 ([13]):

1. A fuzzy class of order 0 is a fuzzy set; it is characterized by a degree of membership of order 1 and a mapping into a one dimensional space.
2. A fuzzy class of order 1 is a fuzzy class; that is, set of fuzzy sets. It is characterized by a pure complex degree of membership. Alternatively, it can be characterized by a degree of membership of order two and a mapping into a two dimensional space.
3. A fuzzy class of order N is a fuzzy set of fuzzy classes of order $N - 1$; it is characterized by a degree of membership of order $N + 1$ and a mapping into an $(N + 1)$ -dimensional space.

Table 11.1 Basic propositional fuzzy logic connectives

Operation	Interpretation
Negation	$f('P) = 1 + j1 - f(P)$
Implication	$f(P \to Q) = \min(1, 1 - p_R + q_R) + j \times \min(1, 1 - p_I + q_I)$
Conjunction	$f(P \otimes Q) = \min(p_R, q_R) + j \times \min(p_I, q_I)$
Disjunction	$f(P \oplus Q) = \min(p_R, q_R) + j \times \min(p_I, q_I)$

Generalized Complex Fuzzy Logic

A general form of a complex fuzzy proposition is: "$x \ldots A \ldots B \ldots$" where A and B are values assigned to linguistic variables and '...' denotes natural language constants. A complex fuzzy proposition P can get any pair of truth values from the Cartesian interval $[0, 1] \times [0, 1]$ or the unit circle. Formally a fuzzy interpretation of a complex fuzzy proposition P is an assignment of fuzzy truth value of the form $p_r + jp_i$, or of the form $r(p)e^{j\theta(p)}$, to P. In this case, assuming a proposition of the form "$x \ldots A \ldots B \ldots$," then p_r $(r(p))$ is assigned to the term A and p_i $(\theta(p))$ is assigned to the term B.

For example, under one interpretation, the complex fuzzy truth value associated with the complex proposition: "x is a *destructive hurricane with high surge*." can be $0.1 + j0.5$. Alternatively, in another context, the same proposition can be interpreted as having the complex truth value $0.3e^{j0.2}$. As in the case of traditional propositional fuzzy logic we use the tight relation between complex fuzzy classes/complex fuzzy membership to determine the interpretation of connectives. For example, let C denote the complex fuzzy set of "destructive hurricanes with high surge," and let $f_C = c_r + jc_i$, be a specific fuzzy membership function of C, then f_C can be used as the basis for interpretations of P. Next we define several connectives along with their interpretation.

Table 11.1 includes a specific definition of connectives along with their interpretation. In this table P, Q, and S denote complex fuzzy propositions and $f(S)$ denotes the complex fuzzy interpretation of S. We use the fuzzy Łukasiewicz logical system as the basis for the definitions [16, 19]. Hence, the max t-norm is used for conjunction and the min t-conorm is used for disjunction. Nevertheless, other logical systems such as Gödel fuzzy systems can be used [19, 21].

The same axioms used for fuzzy logic are used for complex fuzzy logic, and Modus ponens is the rule of inference.

Complex Fuzzy Propositions and Connectives Examples

Consider the following propositions (P and Q respectively):

1. "x is a *destructive hurricane with high surge*."
2. "x is a *destructive huricane with fast moving center*."

Let A be the term "*destructive hurricane*," Hence, P is of the form: "x is A in B," and Q is of the form " x is A in C. " In this case, the terms "*destructve hurricane*," "*high surge*," and "*fast moving center*", are values assigned to the linguistic variables $\{A, B, C\}$. Furthermore, the term "*destructve hurricane*," can get fuzzy truth values (between 0 and 1) or fuzzy linguistic values such as "*catastriphic*," "*devastating*," "*and disastrous*." Assume that the complex fuzzy interpretation (i.e., degree of confidence or complex fuzzy truth value) of P is $p_r + jp_i$, while the complex fuzzy interpretation of Q is $q_r + jq_i$ $(q_r = p_r)$. Thus, the truth value of "*x is a devastating hurricane*," is p_R, the truth value of "*x is in a high surge*," is p_i, the truth value of "x is a *catastriphic huricane*," is q_r, and the truth value of "x is a *fast moving center*," is q_i, Suppose that the term "*moderate*" stands for "*non* − destructive" which stands for "*NOT destructive*," the term "*low*" stands for "*NOT high*," and the term "*slow*" stands for "*NOT fast*." In this context, *NOT* is interpreted as the fuzzy negation operation. Note that this is not the only way to define these linguistic terms and it is used to exemplify the expressive power and the inference power of the logic. Then, the complex fuzzy interpretation of the following composite propositions is:

1. $f('P) = (1 - p_r) + j(1 - p_I)$

That is, $'P$ denotes the proposition "*x is a non − destructive hurricane with a low surge*." The confidence level in $'P$ is $(1 - p_r) + j(1 - p_i)$; where the fuzzy truth value of the term "*x is a non − destructive hurricane*," is $(1 - p_r)$ and the fuzzy truth value of the term "low surge," is $(1 - p_i)$

2. $'P \to 'Q = \min(1, q_r - p_r) + j \times \min(1, q_i - p_I)$

Thus, $('P \to 'Q)$ denotes the proposition *If* "x is a *non − destructive* hurricane with a low surge,"

THEN x is a *non − destructive huricane with low moving center*." The truth values of individual terms, as well as the truth value of $'P \to 'Q$ are calculated according to Table 11.1.

3. $f(P \oplus 'Q) = \max(p_r, 1 - q_r) + j \times \max(p_i, 1 - q_i)$

That is, $(P \oplus 'Q)$ denotes a proposition such as "x is a *destructive hurricane with high surge*." OR

"x is a *non − destructive huricane with low moving center*." The truth values of individual terms, as well as the truth value of $P \oplus 'Q$ are calculated according to Table 11.1.

4. $f('P \otimes Q) = \min(1 - p_r, q_r) + j \times \min(1 - p_i, q_i)$.

That is, $(P \otimes Q)$ denotes the proposition "*x is a devastating hurricane with high surge*." AND "*x is a devastating huricane with fast moving center*." The truth values of individual terms, as well as the truth value of $'P \otimes Q$ are calculated according to Table 11.1.

Complex Fuzzy Inference Example

Assume that the degree of confidence in the proposition $R = {}'P$ defined above is $r_r + jr_i$, let $S = {}'Q$ and assume that the degree of confidence in the fuzzy implication $T = R \rightarrow S$ is $t_r + jt_i$. Then, using Modus ponens

R
$\underline{R \rightarrow S}$
S

One can infer S with a degree of confidence $\min(r_r, t_r) + j \times \min(r_i, t_i)$.
In other words if one is using:

"*x is a non − destructive hurricane with a low surge*,"
IF "x is a non − destructive hurricane with a low surge," THEN
"x is non − destructive huricane with slow moving center."
"*x is non − destructive huricane with slow moving center*."

Hence, using Modus ponens one can infer:
"x is non − destructive hurricane with slow moving center." with a degree of confidence of $\min(r_r, t_r) + j \times \min(r_i, t_i)$.

11.3.5 Neuro-Fuzzy Systems

The term neuro-fuzzy systems refers to combinations of artificial neural networks and Fuzzy logic. Neuro-fuzzy systems enable modeling human reasoning via fuzzy inference systems along with the modeling of human learning via the learning and connectionist structure of neural networks. Neuro-fuzzy systems can serve as highly efficient mechanisms for inference and learning under uncertainty. Furthermore incremental learning techniques can enable observing outliers and the Fuzzy inference can allow these outliers to coexist (with low degrees of membership) with "main-stream" data. As more information about the outliers becomes available, the information, and the derivatives of the rate of information flow can be used to identify potential black swans that are hidden in the outliers The classical model of Neuro-Fuzzy systems can be extended to include multidimensional Fuzzy logic and inference systems in numerical domains and in domains characterized by linguistic variables. We plan to address this in future research.

11.3.6 Incremental Fuzzy Clustering

Clustering is a widely used mechanism for pattern recognition and classification. Fuzzy clustering (e.g., the Fuzzy C-means) enables patterns to be members of more than one cluster. Additionally, it enables maintaining clusters that represent outliers through low degree of membership. These clusters would be discarded in clustering of hard (vs. fuzzy) data. The incremental and dynamic clustering (e.g., the incremental Fuzzy ISODATA) enable the clusters' structures to change as information is accumulated. Again, this is a strong mechanism for enabling identification of unlikely events (i.e., black swans) without premature discarding of these events. The clustering can be performed in a traditional feature space composed of numerical measurements of feature values. Alternatively, the clustering can be performed in a multidimensional fuzzy logic space where the features represent values of linguistic variables The combination of powerful classification capability, adaptive and dynamic mechanisms, as well as the capability to consider uncertain data, maintain data with low likelihood of occurrence, and use a combination of numerical and linguistic values makes this tools one of the most promising tools for detecting black swans. We are currently engaged in research on dynamic and incremental fuzzy clustering and it is evident that the methodology can serve as a highly efficient tool for identifying outliers. We plan to report on this research in the near future.

11.4 Conclusions

In this chapter, we have outlined features of disasters using the metaphor of gray swans. We have shown that an important part of the challenges related to disaster are identifying slow evolving uncertain data that points to the potential of occurrence of disaster before it occurs and fast evolving data concerning the secondary effect of disasters after the occurrence of a major disaster. We have outlined as set of fuzzy logic based tools that can be used to address these and other challenges related to DMM.

While the USA and NATO continue to lead the technical agenda in DMM technologies, recent disasters are showing that there is still a lack of technology-based tools in specific decision support tools for addressing disaster, mitigating their adverse impact and managing disaster response programs. Thus, the USA and NATO must develop additional DMM capabilities. Additional activities that will assist in DMM programs include [22]:

1. Accelerated delivery of technical capabilities for DMM
2. Development of world class science, technology, engineering and mathematics (STEM) capabilities for the DOD and the Nation.

On the top of those important tasks, one should never forget that in the development of DMM programs we do not have the luxury of neglecting human intelligence [23]. In any fuzzy event related to a gray swan an investigation after the fact reveals enough clear data points which have been read correctly but were not treated properly.

In the future, we intend to investigate the DMM utility of several other fuzzy logic based tools including:

1. Value-at-Risk (VaR) under Fuzzy uncertainty
2. Non-cooperative Fuzzy games
3. Fuzzy logic driven web crawlers and web-bots
4. Fuzzy Expert Systems and Fuzzy Dynamic Forecasting (FDEs)

Finally, we plan to expand our research on complex fuzzy logic based neuro-fuzzy systems as well as the research on incremental and dynamic fuzzy clustering. Both of these research threads are expected to provide significant advancement to our capability to identify and neutralized (as much as possible) first generation and second generation gray swans.

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